```
> with(ListTools):
  K:=array(1..142, [351, 336, 304, 291, 315, 270, 328, 283, 275, 273, 273,
  284, 250, 280, 283, 270, 226, 260, 264, 232, 247, 228, 228, 203, 227, 214
  196, 211, 192, 219, 163, 194, 204, 162, 187, 181, 150, 184, 147, 171, 166
  150, 145, 142, 153, 146, 150, 136, 138, 127, 130, 119, 139, 120, 102, 127
  110, 100, 98, 108, 100, 94, 97, 117, 112, 102, 78, 85, 87, 96, 100, 95,
  72, 85, 79, 73, 88, 87, 78, 96, 78, 91, 58, 71, 66, 69, 74, 66, 60, 89,
  57, 59, 67, 60, 52, 56, 55, 55, 53, 54, 63, 58, 58, 50, 56, 45, 48, 38,
  37, 47, 48, 41, 42, 56, 52, 36, 38, 35, 37, 34, 34, 34, 29, 34, 35, 32,
  37, 34, 34, 37, 34, 34, 26, 37, 37, 26, 30, 30, 36, 31, 30, 26]):
```

```
Warning, the assigned name Group now has a global binding
```

```
> rechts:=(1/sum(K[i],
  i=1..N)*sum(K[i]*t[i],i=1..N)+(T*exp(-T/tau))/(1-exp(-T/tau)));
```

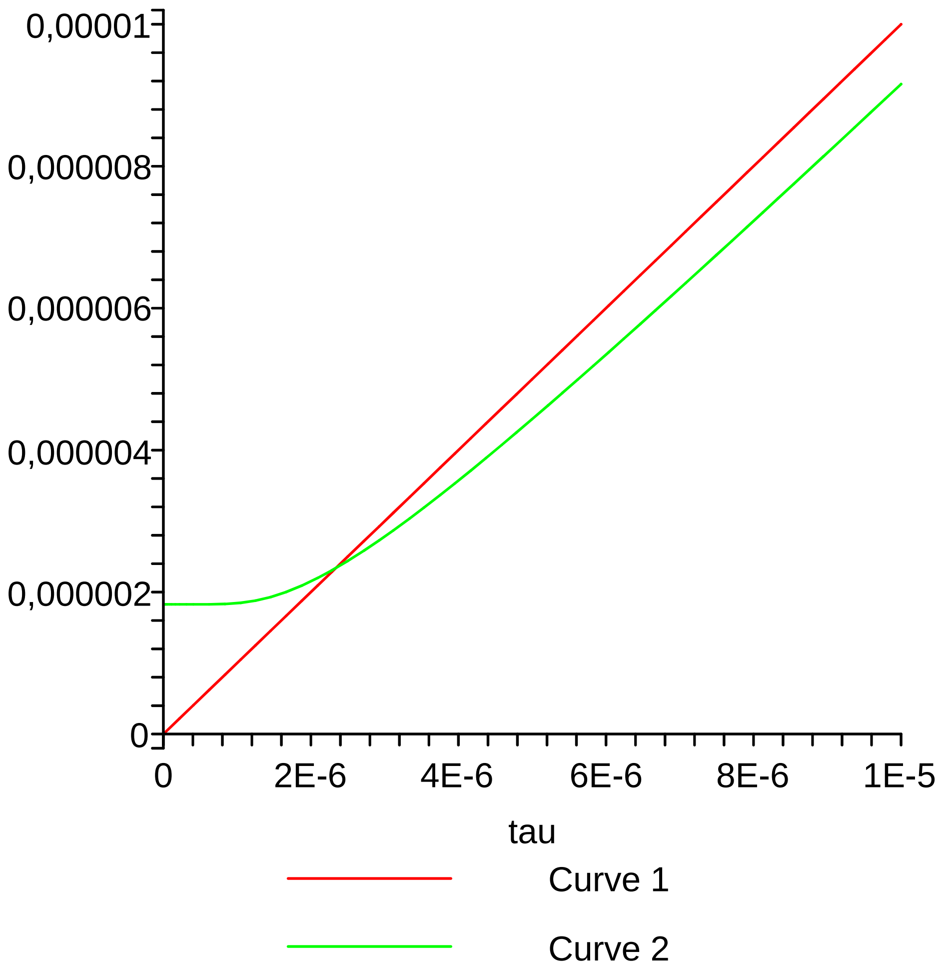
$$rechts := \frac{\sum_{i=1}^{N} K_i\, t_i}{\sum_{i=1}^{N} K_i} + \frac{T\, \mathbf{e}^{\left(-\frac{T}{\tau}\right)}}{1 - \mathbf{e}^{\left(-\frac{T}{\tau}\right)}}$$

```
> Werte:=[KS=9, KE=150, k=41.7e-9]:
  Werte:=Flatten([Werte, eval([N=1+KE-KS, T=k*(1+KE - KS)], Werte)]):
  Werte:=Flatten([Werte, eval([t=[seq((i)*k,i=1..eval(N, Werte))]],
  Werte)]);
```

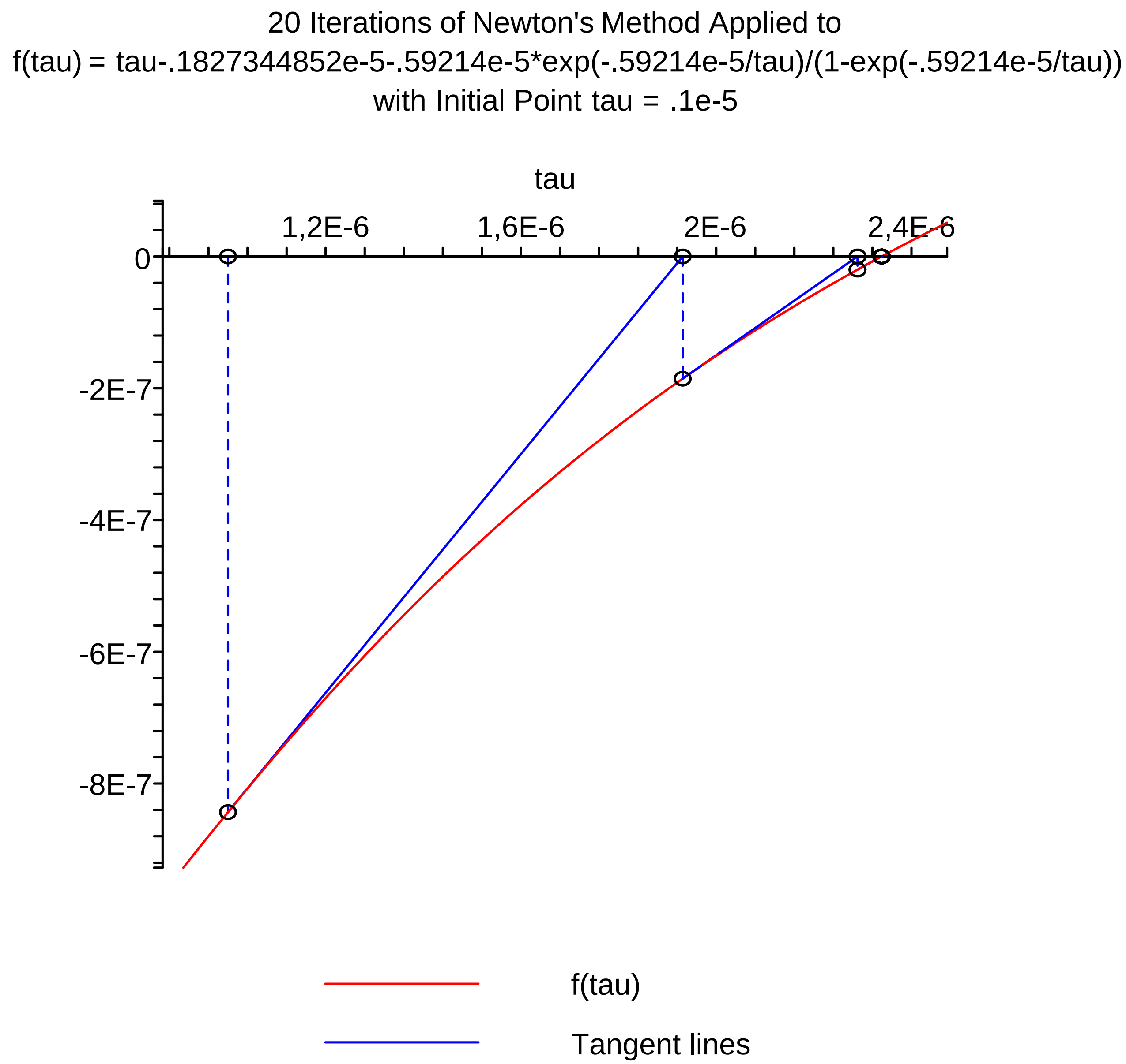
$Werte$:= [KS = 9, KE = 150, k = 4.17 10^{-8}, N = 142, T = 0.0000059214, t = [4.17 10^{-8}, 8.34 10^{-8}, 1.251 10^{-7},
1.668 10^{-7}, 2.085 10^{-7}, 2.502 10^{-7}, 2.919 10^{-7}, 3.336 10^{-7}, 3.753 10^{-7}, 4.170 10^{-7}, 4.587 10^{-7}, 5.004 10^{-7},
5.421 10^{-7}, 5.838 10^{-7}, 6.255 10^{-7}, 6.672 10^{-7}, 7.089 10^{-7}, 7.506 10^{-7}, 7.923 10^{-7}, 8.340 10^{-7}, 8.757 10^{-7},
9.174 10^{-7}, 9.591 10^{-7}, 0.0000010008, 0.0000010425, 0.0000010842, 0.0000011259, 0.0000011676,
0.0000012093, 0.0000012510, 0.0000012927, 0.0000013344, 0.0000013761, 0.0000014178,
0.0000014595, 0.0000015012, 0.0000015429, 0.0000015846, 0.0000016263, 0.0000016680,
0.0000017097, 0.0000017514, 0.0000017931, 0.0000018348, 0.0000018765, 0.0000019182,
0.0000019599, 0.0000020016, 0.0000020433, 0.0000020850, 0.0000021267, 0.0000021684,
0.0000022101, 0.0000022518, 0.0000022935, 0.0000023352, 0.0000023769, 0.0000024186,
0.0000024603, 0.0000025020, 0.0000025437, 0.0000025854, 0.0000026271, 0.0000026688,
0.0000027105, 0.0000027522, 0.0000027939, 0.0000028356, 0.0000028773, 0.0000029190,
0.0000029607, 0.0000030024, 0.0000030441, 0.0000030858, 0.0000031275, 0.0000031692,
0.0000032109, 0.0000032526, 0.0000032943, 0.0000033360, 0.0000033777, 0.0000034194,
0.0000034611, 0.0000035028, 0.0000035445, 0.0000035862, 0.0000036279, 0.0000036696,
0.0000037113, 0.0000037530, 0.0000037947, 0.0000038364, 0.0000038781, 0.0000039198,
0.0000039615, 0.0000040032, 0.0000040449, 0.0000040866, 0.0000041283, 0.0000041700,
0.0000042117, 0.0000042534, 0.0000042951, 0.0000043368, 0.0000043785, 0.0000044202,
0.0000044619, 0.0000045036, 0.0000045453, 0.0000045870, 0.0000046287, 0.0000046704,
0.0000047121, 0.0000047538, 0.0000047955, 0.0000048372, 0.0000048789, 0.0000049206,
0.0000049623, 0.0000050040, 0.0000050457, 0.0000050874, 0.0000051291, 0.0000051708,
0.0000052125, 0.0000052542, 0.0000052959, 0.0000053376, 0.0000053793, 0.0000054210,
0.0000054627, 0.0000055044, 0.0000055461, 0.0000055878, 0.0000056295, 0.0000056712,
0.0000057129, 0.0000057546, 0.0000057963, 0.0000058380, 0.0000058797, 0.0000059214]]

```
> plot([
```

```
    tau,
    eval(rechts, Werte)
 ],tau=0..1e-5);
```




```
> Student[Calculus1][NewtonsMethod](tau-eval(rechts, Werte),tau=1e-6, outpu
  = plot, iterations = 20 ); tau=
  Student[Calculus1][NewtonsMethod](tau-eval(rechts, Werte),tau=1e-6,
  iterations = 20 );
```



$$\tau = 0.000002338943195$$

```
> Fehler:=2*sigma=2*sqrt(sum(K[i]*t[i]^2,i=1..N))/sum(K[i],i=1..N);
  eval(Fehler, Werte);
```

$$Fehler := 2\,\sigma = \frac{2\sqrt{\sum_{i=1}^{N} K_i\, t_i^{2}}}{\sum_{i=1}^{N} K_i}$$

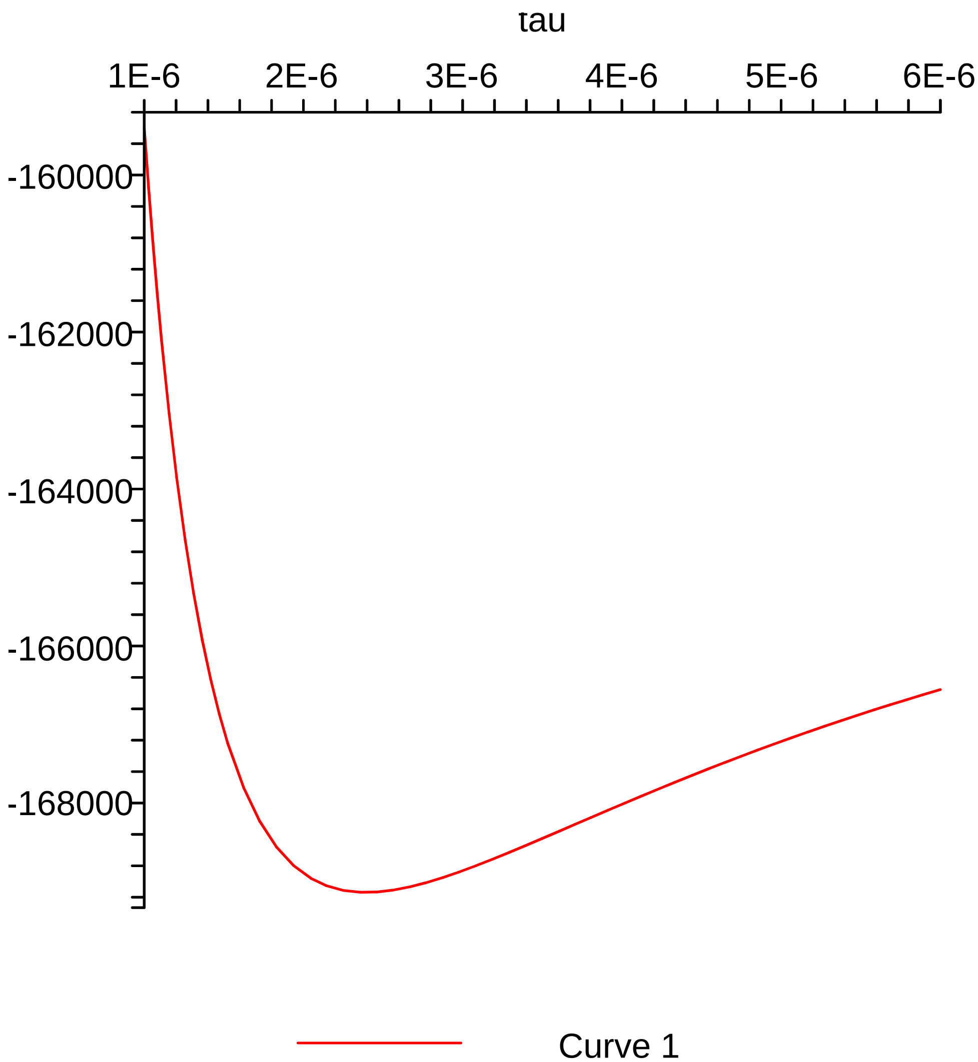
$$2\,\sigma = 3.618227320\ 10^{-8}$$

```
> # 3.       Anwendung der Likelihoodmethode auf die Poissonverteilung
  f:=tau->-2*sum(K[i]*ln(int(1/tau*sum(K[i],i=1..N)/(1-exp(-N*k/tau))*exp(-
  /tau),t=t[i]..t[i]+k)),i=1..N);
```

```
plot(eval([f_taylor(tau), f(tau)],Werte),tau=1e-6..6e-6);
```

$$f := \tau \rightarrow -2 \left(\sum_{i=1}^{N} K_i \ln \left(\int_{t_i}^{t_i + k} \frac{\left(\sum_{i=1}^{N} K_i \right) e^{\left(-\frac{t}{\tau} \right)}}{\tau \left(1 - e^{\left(-\frac{N k}{\tau} \right)} \right)} \, dt \right) \right)$$

```
Warning, unable to evaluate 1 of the 2 functions to numeric values in the region; see the
plotting command's help page to ensure the calling sequence is correct
```




```
> (taylor(eval(f(tau), Werte),tau=2e-6,7));
```

$$-1.688969739\ 10^5 - 1.449889550\ 10^9\ (\tau - 0.000002) + 2.817939251\ 10^{15}\ (\tau - 0.000002)^2 - 2.214472057\ 10^{21}\ (\tau - 0.000002)^3 + 1.393498346\ 10^{27}\ (\tau - 0.000002)^4 - 7.887165541\ 10^{32}\ (\tau - 0.000002)^5 + 4.199201573\ 10^{38}\ (\tau - 0.000002)^6 + O((\tau - 0.000002)^7)$$

```
> # von oben abgeschrieben
  f_taylor:=tau->-168896.9739-1449889819.*(tau-.2e-5)+.2817938767e16*(tau-.
  e-5)^2-.2214471952e22*(tau-.2e-5)^3+.1393498768e28*(tau-.2e-5)^4-.788715
  17e33*(tau-.2e-5)^5+.4199199905e39*(tau-.2e-5)^6:
```

```
> # find Minimum
  with(Optimization):tau_minimal:=NLPSolve(eval(f(tau),Werte),
  tau=1e-6..6e-6, initialpoint=[tau=2e-6]);
  solve(f_taylor(tau)=tau_minimal[1]+1); # ohne Taylor leider kein Ergebnis
```

$$tau\_minimal := [-1.69138833512532874\ 10^5, [\tau = 0.00000239304920498363138]]$$

$$0.000001574003215 - 0.000001485065134\ I, 0.000001574003215 + 0.000001485065134\ I, 0.000002365044716, 0.000002415541814, 0.000002974829877 - 7.961684290\ 10^{-7}\ I, 0.000002974829877 + 7.961684290\ 10^{-7}\ I$$

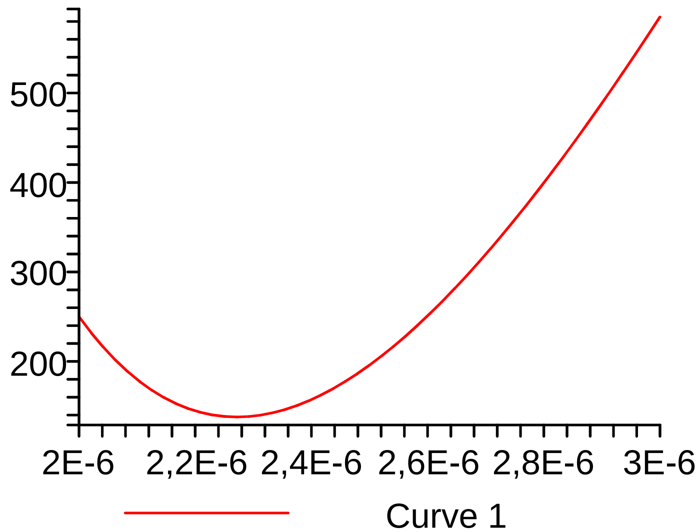
```
> #die reellen Lösungen herauspicken
  delta_tau=max(abs(.2415541814e-5-.239304920498343402e-5),
  abs(.2365044716e-5-.239304920498343402e-5));
```

$$delta\_tau = 2.8004489\ 10^{-8}$$

```
> # Anwendung der Likelihoodmethode auf die Gaußverteilung
  xi_sqare:=tau->sum((K[i]-int(1/tau*sum(K[j],j=1..N)/(1-exp(-N*k/tau))*exp
  -t/tau),t=t[i]..t[i]+k))^2/K[i],i=1..N);
```

$$xi\_sqare := \tau \rightarrow \sum_{i=1}^{N} \frac{\left(K_i - \int_{t_i}^{t_i + k} \frac{\left(\sum_{j=1}^{N} K_j\right) \mathbf{e}^{\left(-\frac{t}{\tau}\right)}}{\tau \left(1 - \mathbf{e}^{\left(-\frac{N k}{\tau}\right)}\right)} dt\right)^2}{K_i}$$

```
> plot(eval(xi_sqare(tau), Werte), tau=2e-6..3e-6);
>
```




```
> tau_minimum=NLPSolve(eval(xi_sqare(tau), Werte), tau=2e-6..3e-6);
```

$$tau\_minimum = [137.866499351056178, [\tau = 0.00000226880850145564868]]$$

```
> #solve(eval(xi_sqare(tau), Werte)=137.866499351056148+1);
```

```
> #die reellen Lösungen herauspicken
  delta_tau=max(abs(0.2374518952e-5-0.239999999999999990e-5),
  abs(0.2441899465e-5-0.239999999999999990e-5));
```

$$delta\_tau = 4.1899465\ 10^{-8}$$

```
>
>
>
>
>
```