```
> with(ListTools):
  K:=array(1..18, [2478, 2188, 1888, 1616, 1386, 1188, 1002, 824, 755, 658,
  573, 500, 446, 360, 330, 269, 265, 243]);
```

```
Warning, the assigned name Group now has a global binding
```

$$K := [2478, 2188, 1888, 1616, 1386, 1188, 1002, 824, 755, 658, 573, 500, 446, 360, 330, 269, 265, 243]$$

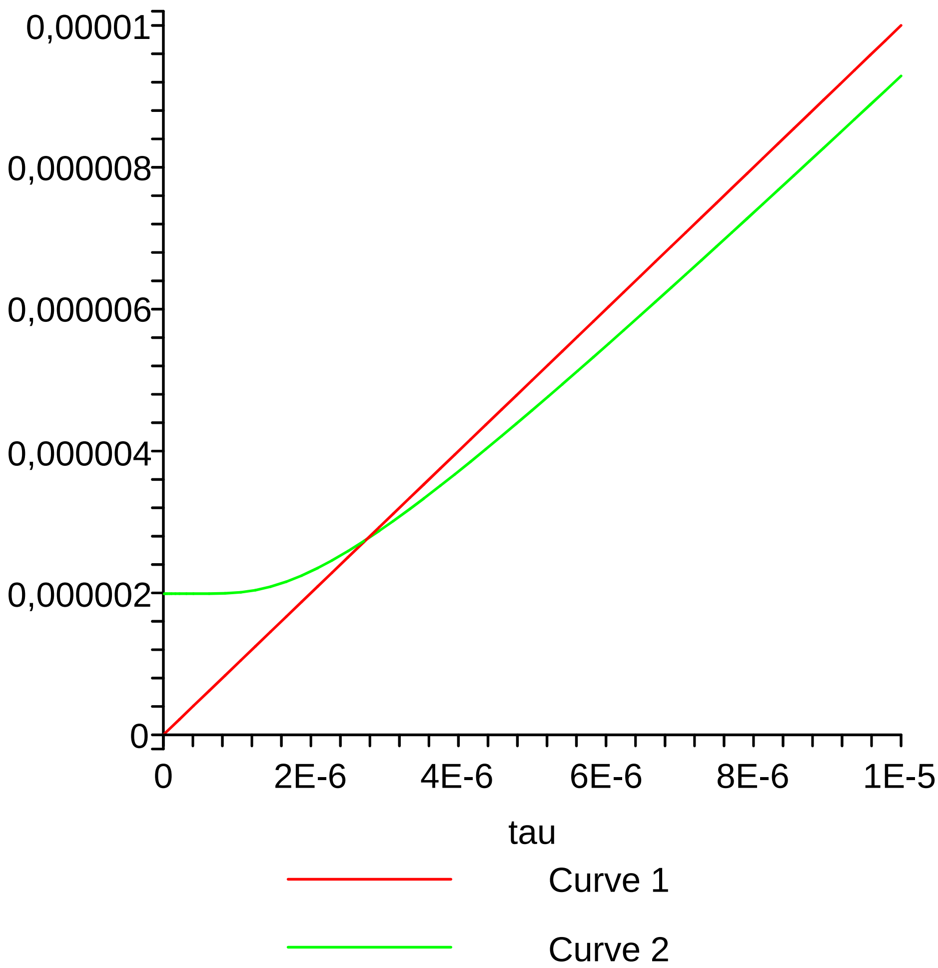
```
> rechts:=(1/sum(K[i],
  i=1..N)*sum(K[i]*t[i],i=1..N)+(T*exp(-T/tau))/(1-exp(-T/tau)));
```

$$rechts := \frac{\sum_{i=1}^{N} K_i\, t_i}{\sum_{i=1}^{N} K_i} + \frac{T\, \mathbf{e}^{\left(-\frac{T}{\tau}\right)}}{1 - \mathbf{e}^{\left(-\frac{T}{\tau}\right)}}$$

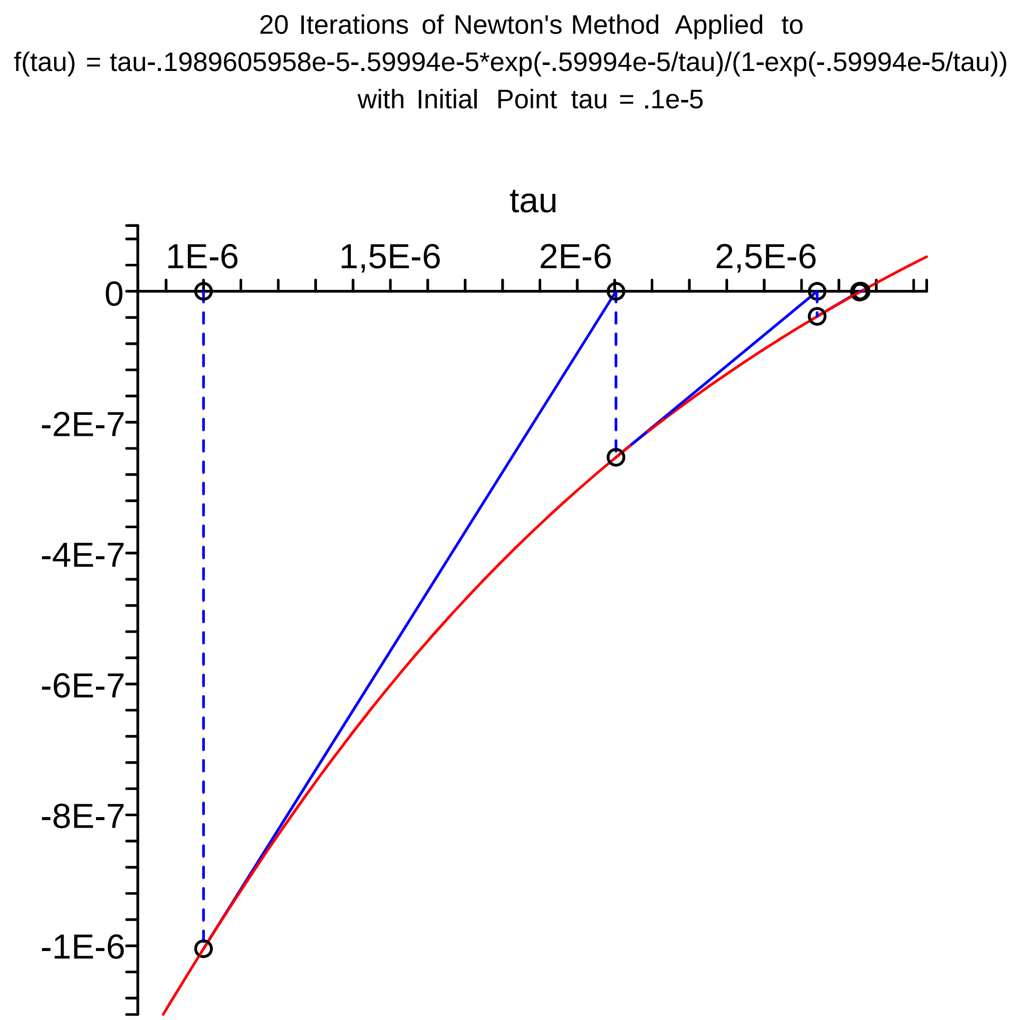
```
> Werte:=[KS=2, KE=19, k=333.3e-9]:
  Werte:=Flatten([Werte, eval([N=1+KE-KS, T=k*(1+KE - KS)], Werte)]):
  Werte:=Flatten([Werte, eval([t=[seq((i)*k,i=1..eval(N, Werte))]],
  Werte)]);
```

$Werte := [KS = 2, KE = 19, k = 3.333\ 10^{-7}, N = 18, T = 0.0000059994, t = [3.333\ 10^{-7}, 6.666\ 10^{-7}, 9.999\ 10^{-7},$
$0.0000013332, 0.0000016665, 0.0000019998, 0.0000023331, 0.0000026664, 0.0000029997,$
$0.0000033330, 0.0000036663, 0.0000039996, 0.0000043329, 0.0000046662, 0.0000049995,$
$0.0000053328, 0.0000056661, 0.0000059994]]$

```
> plot([
      tau,
      eval(rechts, Werte)
  ],tau=0..1e-5);
```




```
> Student[Calculus1][NewtonsMethod](tau-eval(rechts, Werte),tau=1e-6, outpu
  = plot, iterations = 20 );
  tau=Student[Calculus1][NewtonsMethod](tau-eval(rechts, Werte),tau=1e-6,
  iterations = 20 );
```



$\tau = 0.000002758976536$

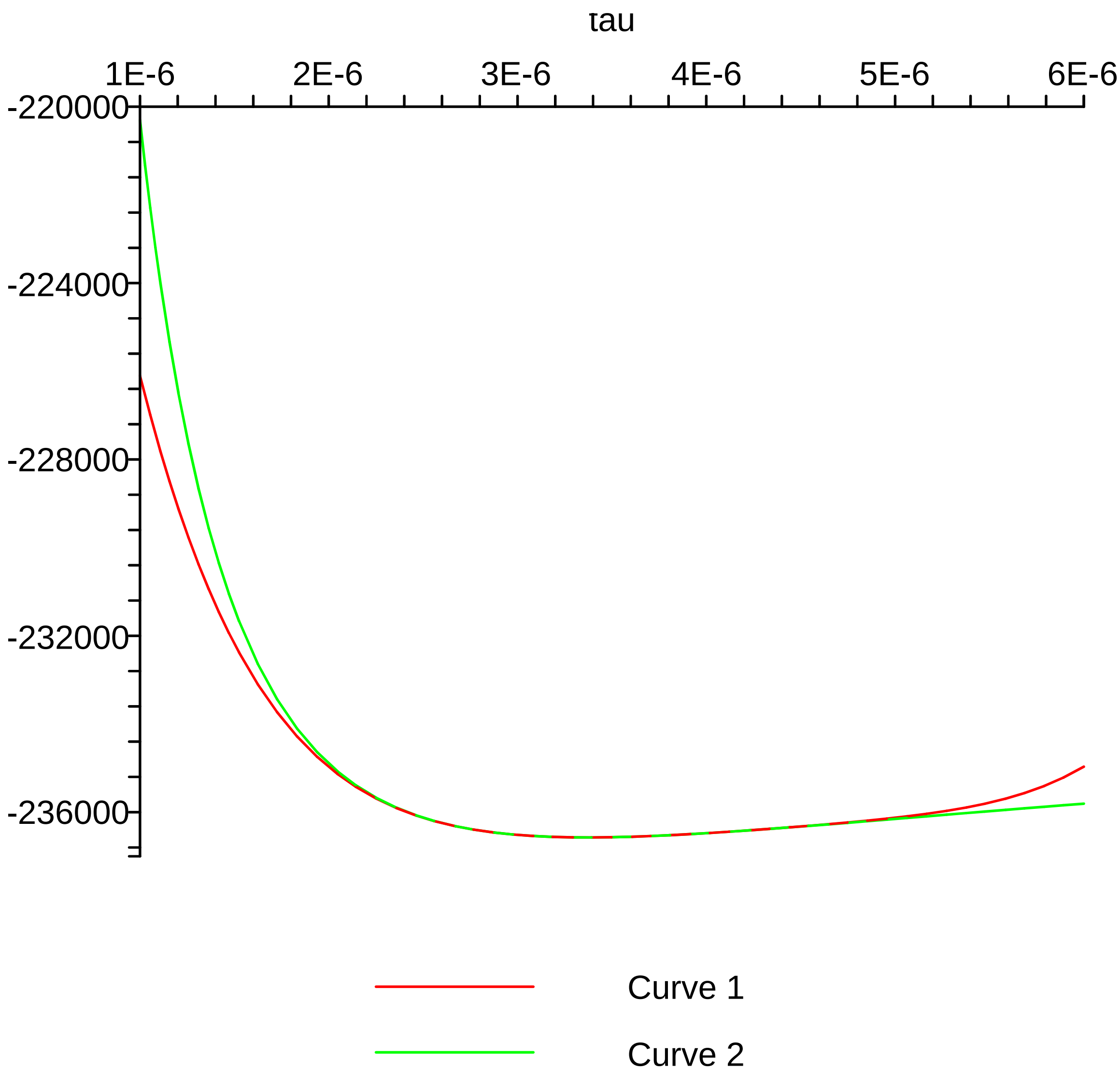
```
> Fehler:=2*sigma=2*sqrt(sum(K[i]*t[i]^2,i=1..N))/sum(K[i],i=1..N);
  eval(Fehler, Werte);
```

$$Fehler := 2\,\sigma = \frac{2\sqrt{\sum_{i=1}^{N} K_i\, t_i^2}}{\sum_{i=1}^{N} K_i}$$

$$2\,\sigma = 3.820768508\; 10^{-8}$$

```
> # 3.        Anwendung der Likelihoodmethode auf die Poissonverteilung
  f:=tau->-2*sum(K[i]*ln(int(1/tau*sum(K[j],j=1..N)/(1-exp(-N*k/tau))*exp(-
  /tau),t=t[i]..t[i]+k)),i=1..N);
  plot(eval([f_taylor(tau), f(tau)],Werte),tau=1e-6..6e-6, -220e3..-237e3);
```

$$f := \tau \to -2 \left(\sum_{i=1}^{N} K_i \ln \left(\int_{t_i}^{t_i + k} \frac{\left(\sum_{j=1}^{N} K_j \right) \mathbf{e}^{\left(-\frac{t}{\tau} \right)}}{\tau \left(1 - \mathbf{e}^{\left(-\frac{N k}{\tau} \right)} \right)} dt \right) \right)$$




```
> taylor(eval(f(tau), Werte),tau=3.5e-6,7);
```

$$-2.365673416\ 10^{5} + 7.85363732\ 10^{7} (\tau - 0.0000035) + 2.711225430\ 10^{14} (\tau - 0.0000035)^{2} - 1.537581854\ 10^{20} (\tau - 0.0000035)^{3} + 6.325697527\ 10^{25} (\tau - 0.0000035)^{4} - 2.283363405\ 10^{31} (\tau - 0.0000035)^{5} + 7.659923486\ 10^{36} (\tau - 0.0000035)^{6} + O((\tau - 0.0000035)^{7})$$

```
> # von oben abgeschrieben
  f_taylor:=tau->-236567.3416+78536373.2*(tau-0.35e-5)+0.2711225430e15*(tau
```

```
0.35e-5)^2-0.1537581854e21*(tau-0.35e-5)^3+0.6325697527e26*(tau-0.35e-5)^
-0.2283363405e32*(tau-0.35e-5)^5+0.7659923486e37*(tau-0.35e-5)^6:
```

```
> # find Minimum
  with(Optimization):tau_minimal:=NLPSolve(eval(f(tau),Werte),
  tau=1e-6..6e-6, initialpoint=[tau=2e-6]);
  solve(f_taylor(tau)=tau_minimal[1]+1); # ohne Taylor leider kein Ergebnis
```

$$tau\_minimal := [-2.36572610329328076\ 10^5, [\tau = 0.00000336778267315261830]]$$

$0.000003047643051 - 0.000002554404931\ I,\ 0.000003047643051 + 0.000002554404931\ I,$
$0.000003316852060,\ 0.000003425833826,\ 0.000005571474981 - 0.000001343651059\ I,$
$0.000005571474981 + 0.000001343651059\ I$

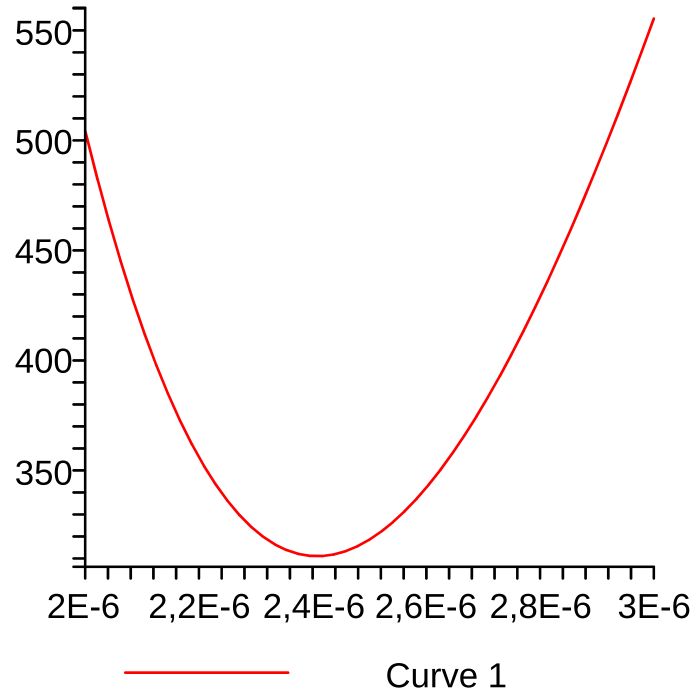
```
> #die reellen Lösungen herauspicken
  delta_tau=max(abs(0.3316852060e-5-0.336778267315261830e-5),
  abs(0.3425833826e-5-0.336778267315261830e-5));
```

$$delta\_tau = 5.8051153\ 10^{-8}$$

```
> # Anwendung der Likelihoodmethode auf die Gaußverteilung
  xi_sqare:=tau->sum((K[i]-int(1/tau*sum(K[j],j=1..N)/(1-exp(-N*k/tau))*exp
  -t/tau),t=t[i]..t[i]+k))^2/K[i],i=1..N);
```

$$xi\_sqare := \tau \to \sum_{i=1}^{N} \frac{\left(K_i - \int_{t_i}^{t_i+k} \frac{\left(\sum_{j=1}^{N} K_j\right) \mathbf{e}^{\left(-\frac{t}{\tau}\right)}}{\tau \left(1 - \mathbf{e}^{\left(-\frac{N k}{\tau}\right)}\right)}\, dt\right)^2}{K_i}$$

```
> plot(eval(xi_sqare(tau), Werte), tau=2e-6..3e-6);
>
```




```
> NLPSolve(eval(xi_sqare(tau), Werte), tau=2e-6..3e-6);
```

$$[311.081990095164144, [\tau = 0.00000239999999999999990]]$$

```
> solve(eval(xi_sqare(tau), Werte)=311.081990095164144+1);
```

$2.743743525\ 10^{-9} + 1.123282330\ 10^{-7}\ I$, $2.743743525\ 10^{-9} - 1.123282330\ 10^{-7}\ I$,
$3.198554675\ 10^{-9} + 1.192966065\ 10^{-7}\ I$, $3.198554675\ 10^{-9} - 1.192966065\ 10^{-7}\ I$,
$3.403358764\ 10^{-9} + 1.271984417\ 10^{-7}\ I$, $3.403358764\ 10^{-9} - 1.271984417\ 10^{-7}\ I$,
$3.895915923\ 10^{-9} + 1.364248382\ 10^{-7}\ I$, $3.895915923\ 10^{-9} - 1.364248382\ 10^{-7}\ I$,
$4.769602394\ 10^{-9} + 1.461929111\ 10^{-7}\ I$, $4.769602394\ 10^{-9} - 1.461929111\ 10^{-7}\ I$,
$5.115455493\ 10^{-9} + 1.592575608\ 10^{-7}\ I$, $5.115455493\ 10^{-9} - 1.592575608\ 10^{-7}\ I$,
$7.248564195\ 10^{-9} + 1.726310467\ 10^{-7}\ I$, $7.248564195\ 10^{-9} - 1.726310467\ 10^{-7}\ I$,
$7.984436565\ 10^{-9} + 1.899181217\ 10^{-7}\ I$, $7.984436565\ 10^{-9} - 1.899181217\ 10^{-7}\ I$,
$8.184812104\ 10^{-8} + 1.627542551\ 10^{-7}\ I$, $8.184812104\ 10^{-8} - 1.627542551\ 10^{-7}\ I$,
$1.338769634\ 10^{-8} + 2.128426281\ 10^{-7}\ I$, $1.338769634\ 10^{-8} - 2.128426281\ 10^{-7}\ I$,
$1.253885518\ 10^{-8} + 2.401544798\ 10^{-7}\ I$, $1.253885518\ 10^{-8} - 2.401544798\ 10^{-7}\ I$,
$1.786299276\ 10^{-8} + 2.781180209\ 10^{-7}\ I$, $1.786299276\ 10^{-8} - 2.781180209\ 10^{-7}\ I$,
$2.039443154\ 10^{-8} + 3.175815914\ 10^{-7}\ I$, $2.039443154\ 10^{-8} - 3.175815914\ 10^{-7}\ I$,
$3.179608346\ 10^{-8} + 3.927486519\ 10^{-7}\ I$, $3.179608346\ 10^{-8} - 3.927486519\ 10^{-7}\ I$,
$4.737258429\ 10^{-8} + 4.764071154\ 10^{-7}\ I$, $4.737258429\ 10^{-8} - 4.764071154\ 10^{-7}\ I$,
$8.686671467\ 10^{-8} + 6.635515986\ 10^{-7}\ I$, $8.686671467\ 10^{-8} - 6.635515986\ 10^{-7}\ I$,
$2.072529812\ 10^{-7} + 9.512579273\ 10^{-7}\ I$, $2.072529812\ 10^{-7} - 9.512579273\ 10^{-7}\ I$, 0.000002441899465,
0.000002374518952

```
> #die reellen Lösungen herauspicken
```

```
delta_tau=max(abs(0.2374518952e-5-0.23999999999999990e-5),
abs(0.2441899465e-5-0.23999999999999990e-5));
```

$$delta\_tau = 4.1899465\ 10^{-8}$$

```
>
```